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Reduction of Alternating Functions to Alternants.

By Wm. Woolsey Johnson.

Denoting by $\phi(a, bcd \dots l)$ any function of the n quantities $a, b, c \dots l$ which is symmetrical with respect to all of the quantities except a, the determinant

$$(1) \qquad \begin{vmatrix} \phi_1(a, bcd \dots l), & \phi_2(a, bcd \dots l), \dots & \phi_n(a, bcd \dots l) \\ \phi_1(b, acd \dots l), & \phi_2(b, acd \dots l), \dots & \phi_n(b, acd \dots l) \\ \vdots & \vdots & \vdots \\ \phi_1(l, abc \dots k), & \phi_2(l, abc \dots k), \dots & \phi_n(l, abc \dots k) \end{vmatrix}$$

is obviously an alternating function of the n quantities.

If each of the functions contains only the leading letter, the determinant takes the form

$$(2) \qquad \begin{vmatrix} f_1(a), & f_2(a), \dots & f_n(a) \\ f_1(b), & f_2(b), \dots & f_n(b) \\ \vdots & \vdots & \vdots \\ f_1(l), & f_2(l), \dots & f_n(l) \end{vmatrix},$$

and is an alternant. The alternant (2) may be represented by means of its principal term

$$(3) \qquad f_1(a) . f_2(b) . f_3(c) \dots f_n(l).$$

Suppose now that the principal term of the alternating function (1) can be separated into parts of the form (3); then will the sum of the alternants represented by these partial terms be equal to the given alternating function. To prove this it is only necessary to notice that, since an interchange of two rows in (1) is equivalent to an interchange of the corresponding letters, any term in (1) may be derived from the principal term by a certain transposition of the letters, and in like manner, the corresponding term in each of the alternants may be derived from its principal term by the same transposition of the letters: thus every term in the expansion of (1) is equal to the sum of the corresponding terms in the expansions of the alternants.

If a determinant of the form (1) be expressed by enclosing its principal term in [ ] with periods separating the several elements, the theorem is that the periods may be dispensed with and the symbol treated as an ordinary algebraic quantity. Thus

$$\begin{vmatrix} bcd, & 1, & a, & a^2 \\ cda, & 1, & b, & b^2 \\ dab, & 1, & c, & c^2 \\ abc, & 1, & d, & d^2 \end{vmatrix} = [bcd.1.c.d^2] = [a^0bc^2d^3] = \zeta^{\frac{1}{2}}(a, b, c, d),$$

the difference product of the quantities a, b, c and d.

When the elements of the alternating function are rational and integral, it may thus always be expressed in terms of simple alternants, that is, alternants of powers; and, since a simple alternant vanishes when two of the exponents are equal, it will frequently happen that many of the parts of an alternating function whose elements are polynomials will vanish. For example,

$$\begin{vmatrix} 1, & b^2 + c^2, & a^2 + bc \\ 1, & c^2 + a^2, & b^2 + ca \\ 1, & a^2 + b^2, & c^2 + ab \end{vmatrix} = [a^0b^0c^4] + [a^2b^0c^2] + [abc^2] + [a^3bc^0],$$

all of which except the last vanish; the result is, therefore,

$$[1.c^2 + a^2.c^2 + ab] = [a^3bc^0] = -[a^0bc^3] = -(a + b + c)\,\zeta^{\frac{1}{2}}(a, b, c).$$